Pseudorandom Quantum States with Proof of Destruction and applications

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Pseudorandom quantum states

Ji-Liu-Song'18

Recall: Pseudorandom States definition

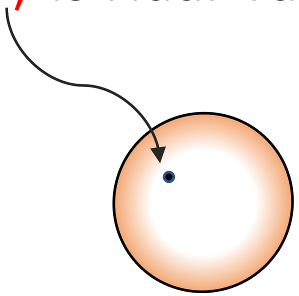
A quantum poly-time (QPT) algorithm G is a **pseudorandom state (PRS) generator** if

- given key $k \in \{0,1\}^\lambda$, $G(k)$ outputs n-qubit state $|\psi_k\rangle$
- for all t, for all poly-time algorithms D (called a **distinguisher**),

$|\psi_k\rangle = G(k)$ for random $k \in \{0,1\}^\lambda$

$$D\left(|\psi_k\rangle^{\otimes t}\right) \approx D\left(|\vartheta\rangle^{\otimes t}\right)$$

$|\vartheta\rangle$ is Haar-random



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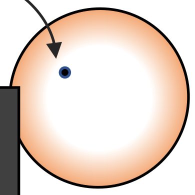

A PRS generator is different from a **state t-design**, where indistinguishability only holds for some *fixed* t.

Pseudorandom *function-like* states

A quantum poly-time algorithm G is a **PRFS generator** if

- given **key** $k \in \{0,1\}^{\lambda}$ and **input** $x \in \{0,1\}^{d}$, $G(k,x)$ outputs n-qubit state $|\psi_{k,x}\rangle$
- for all t, for all distinct inputs $x_1, \dots, x_s$, for all poly-time distinguishers D

$$D\left(|\psi_1\rangle^{\otimes t}, \dots, |\psi_s\rangle^{\otimes t}\right) \quad \approx \quad D\left(|\vartheta_1\rangle^{\otimes t}, \dots, |\vartheta_s\rangle^{\otimes t}\right)$$

$|\psi_i\rangle$'s sampled by:
- sampling random $k \in \{0,1\}^{\lambda}$
- setting $|\psi_i\rangle = G(k, x_i)$ for $i = 1, \dots, s$

$|\vartheta_i\rangle$'s sampled by:
- Independently sampling Haar-random $|\vartheta_i\rangle$ for $i = 1, \dots, s$

Important: the distinguisher D is allowed to depend on $x_1, \dots, x_s$!

Quantum States with Proof of destruction

Motivation

$|\sigma\rangle$

Intrinsic worth. Eg. money

How can Alice verify?

If σ is classical, no hope!

Can you destroy it?

$| \ \rangle$

σ

p

Bank

$p \leftarrow Destruct(|\sigma\rangle)$

✔ ✘ $\leftarrow Verify(k, p)$

$|\sigma\rangle \leftarrow Mint(k)$

Known constructions and comparison to this work

Reference	Based on	Pseudorandomness	Proof of destruction
BS’16, Wie’69, MVW’13, PYJ+’12 CLLZ’21, Shm’22	BB84/Subspace/Coset states	✗	✓

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JLS’21, BS’20	Random phase state	✓	✗

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JLS'21, BS'20	Random phase state	✓	✗
This work	Random phase state on a hidden set	✓	✓

Definitions

Pseudorandom States with proof of destruction (PRSPD)

Keyspace $\{0,1\}^\lambda$ associated with a triplet of efficient algorithms

- $|\psi_k\rangle \leftarrow Gen(k)$
- $p \leftarrow Destruct(|\psi_k\rangle)$
- ✔ ✘ $\leftarrow Ver_k(p)$

Correctness: $\Pr\left[k \leftarrow \{0,1\}^\lambda, |\psi_k\rangle \leftarrow Gen(k), p \leftarrow Destruct(|\psi_k\rangle) : 1 \leftarrow Ver_k(p)\right] = 1.$

Security

- Pseudorandomness
  - Same as the Pseudorandom States
- Unforgeability of proof of destruction

Unforgeability game

p_1

p_2

p_{t+1}

$Ver_k(p_i)$

QPT adversaries

Verification Oracle

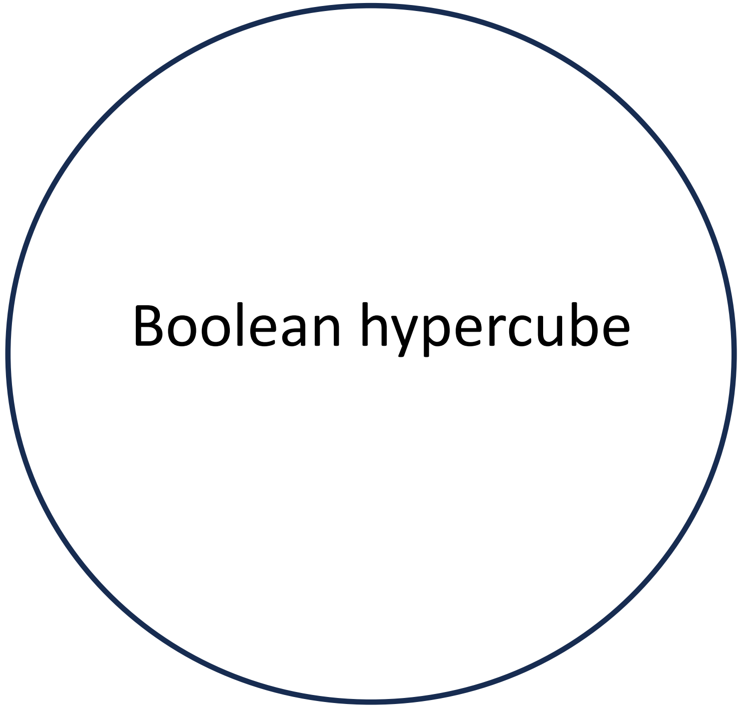
$|\psi_k\rangle^{\otimes t}$

Construction

Construction

Recall Ji-Liu-Song'19 (Simplified by Brakerski-Shmueli'20)

- Pseudorandom function family (PRF): $\{f_k\}_{k \in K}$
- Same keyspace K.



$$|\psi_k\rangle = \frac{1}{2^{\frac{n}{2}}} \sum_x (-1)^{f_k(x)} |x\rangle$$

Random phase state

Sparsifying the construction

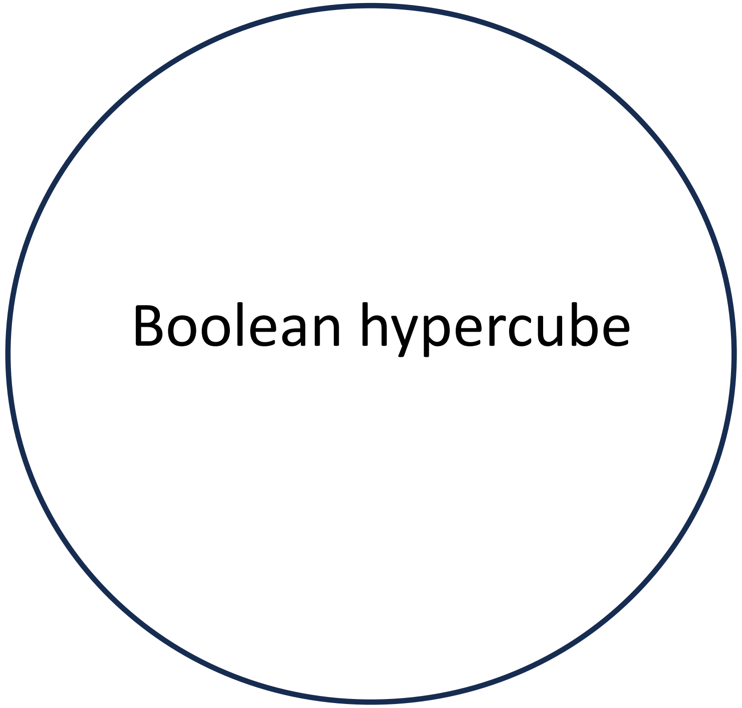
<u>PRS with support on all strings</u>

$$|\psi_k\rangle = \frac{1}{2^{\frac{n}{2}}} \sum_{x} (-1)^{f_k(x)} |x\rangle$$

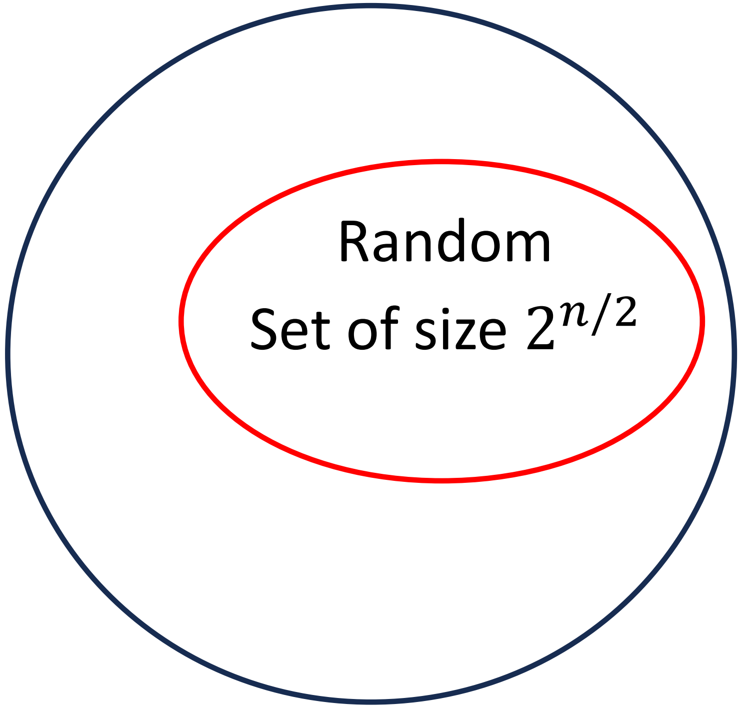
<u>PRSPD with support on a hidden set</u>

$$|\psi_{k,r}\rangle = \frac{1}{2^{\frac{n}{4}}} \sum_{x \in S_r} (-1)^{f_k(x)} |x\rangle$$

S_r : Pseudorandom set of size $2^{n/2}$



Random phase state



Sparsifying the construction

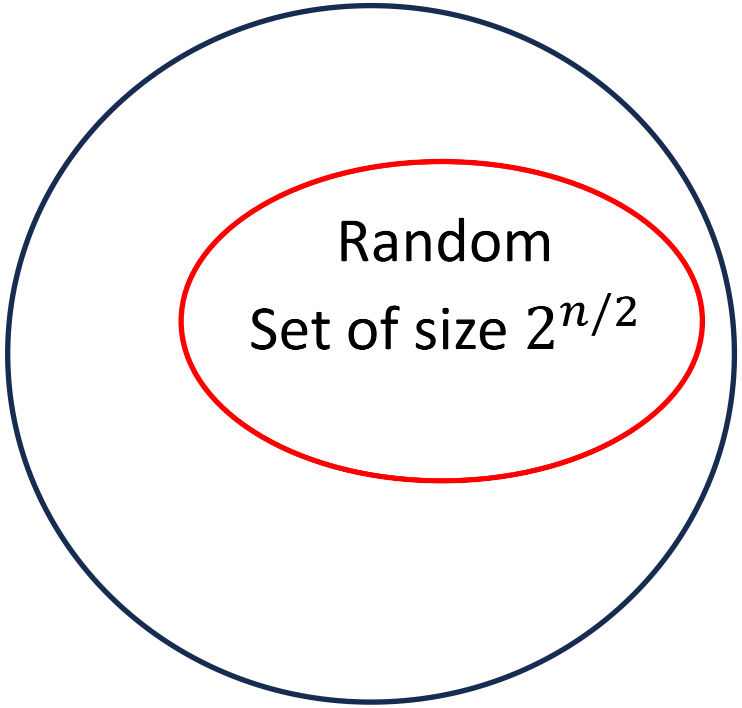
- PRF: $\{f_k\}_{k \in K}$
- Pseudorandom Permutation (PRP): $\{P_r\}_{r \in R}$
- Keyspace $K \times R$

PRSPD with support on a hidden set

$$|\psi_{k,r}\rangle = \frac{1}{2^{\frac{n}{4}}} \sum_{x \in S_r} (-1)^{f_k(x)} |x\rangle$$

$S_r = \{P_r(z) | z \in 0^{n/2} \times \{0,1\}^{\frac{n}{2}}\}$ $\quad r \leftarrow Uniform(R)$ $\quad S_r$: Pseudorandom set of size $2^{n/2}$

Random phase state



- Destruct: computational basis measurement
- $Ver_{k,r}()$: Membership in S_r

Technical lemma for security proofs

$k \leftarrow \{0,1\}^{\lambda}$

$|\psi_k\rangle \leftarrow Gen(k)$

$\vec{x} = x_1, x_2, \ldots x_t \leftarrow \{0,1\}^n$

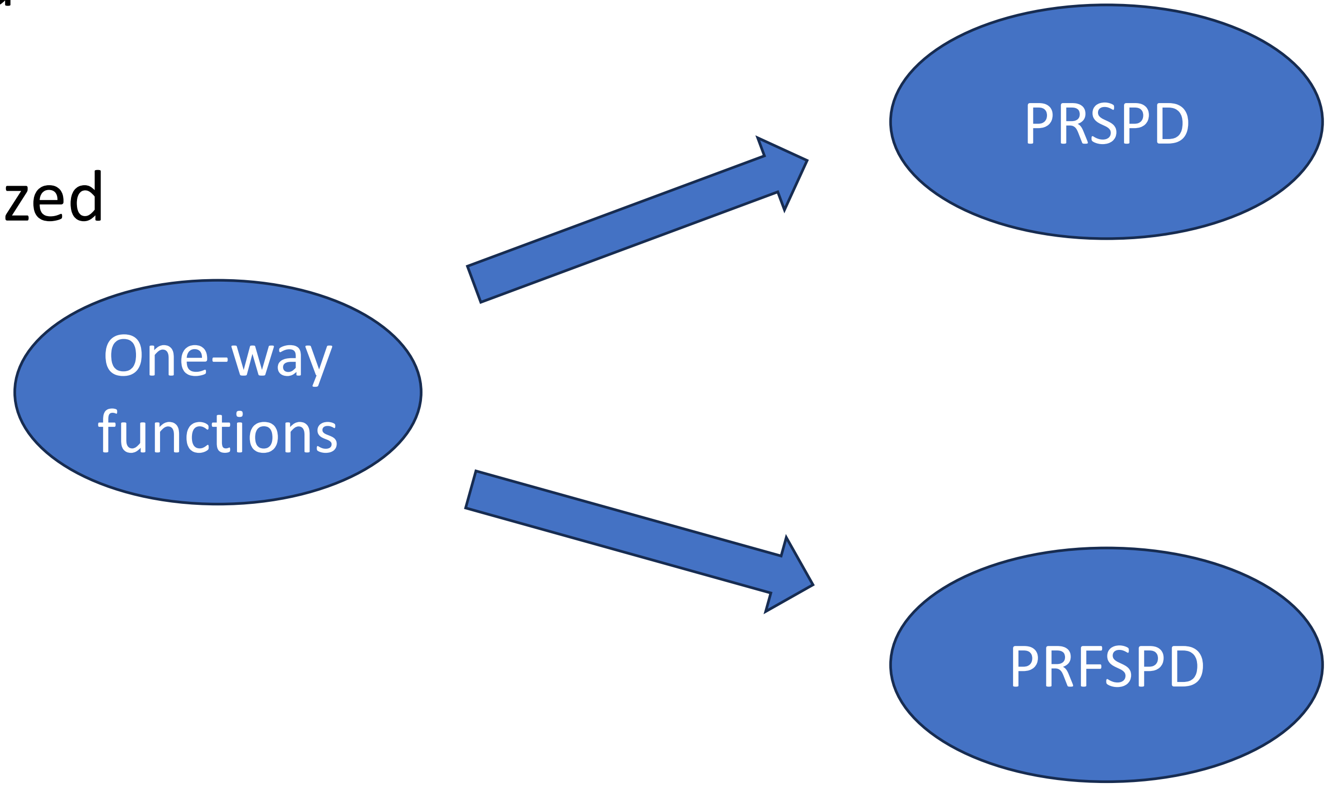
$|Sym_{\vec{x}}\rangle \propto \sum_{\pi} |x_{\pi(1)}, x_{\pi(2)}, \ldots x_{\pi(t)}\rangle$

$Ver_k(\cdot)$ $\approx$ $1_{\{x_1, x_2, \ldots, x_t\}}(\cdot)$

$|\psi_k\rangle^{\otimes t}$ $|Sym_{\vec{x}}\rangle$

Pseudorandom Function-like States with Proof of Destruction (PRFSPD)

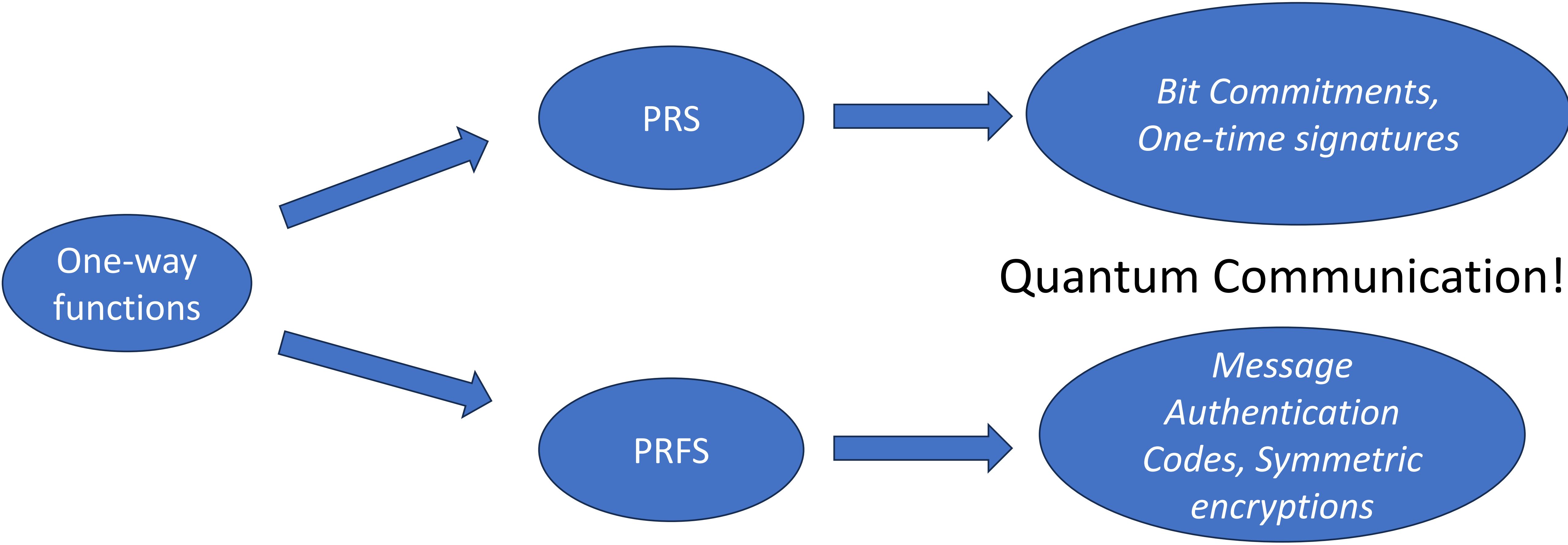
- Definition can be generalized
- Construction can be generalized



Applications

Applications of PRS, PRFS

(Ananth-Qian-Yuen'21, Morimae-Yamakawa'21, etc)

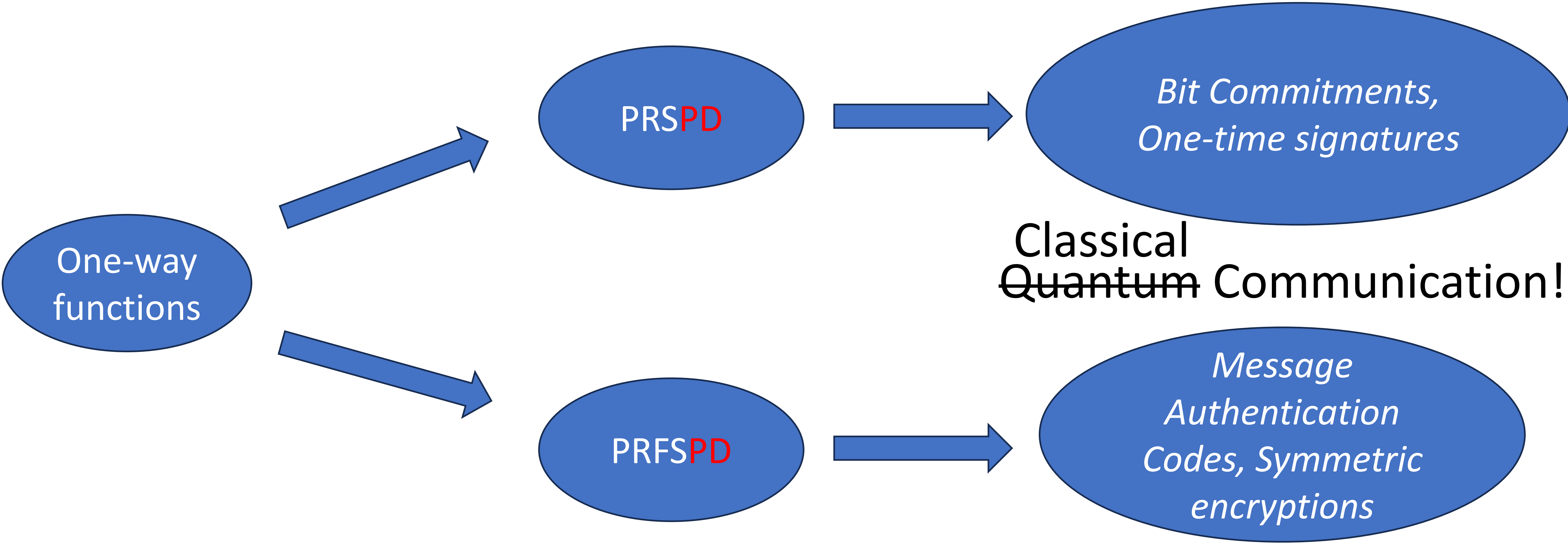


Black box

Necessary?

Applications of PRSPD, PRFSPD

(Our Work)



Black box

Applications of PRSPD, PRFSPD

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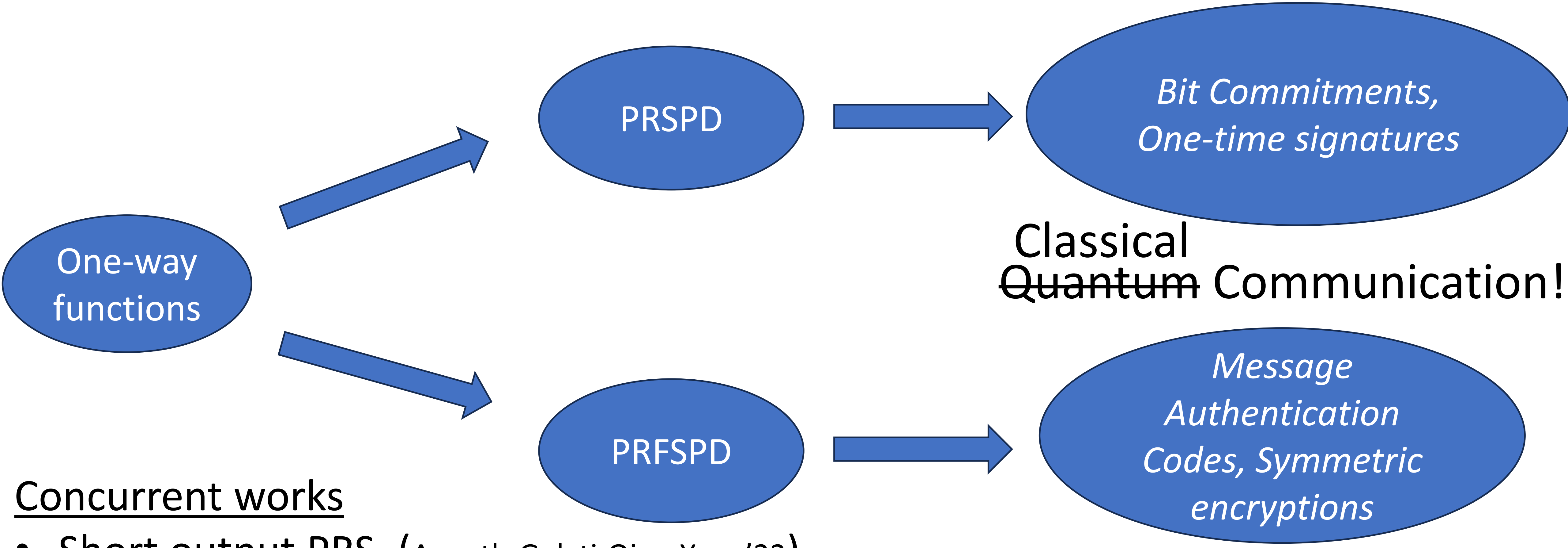
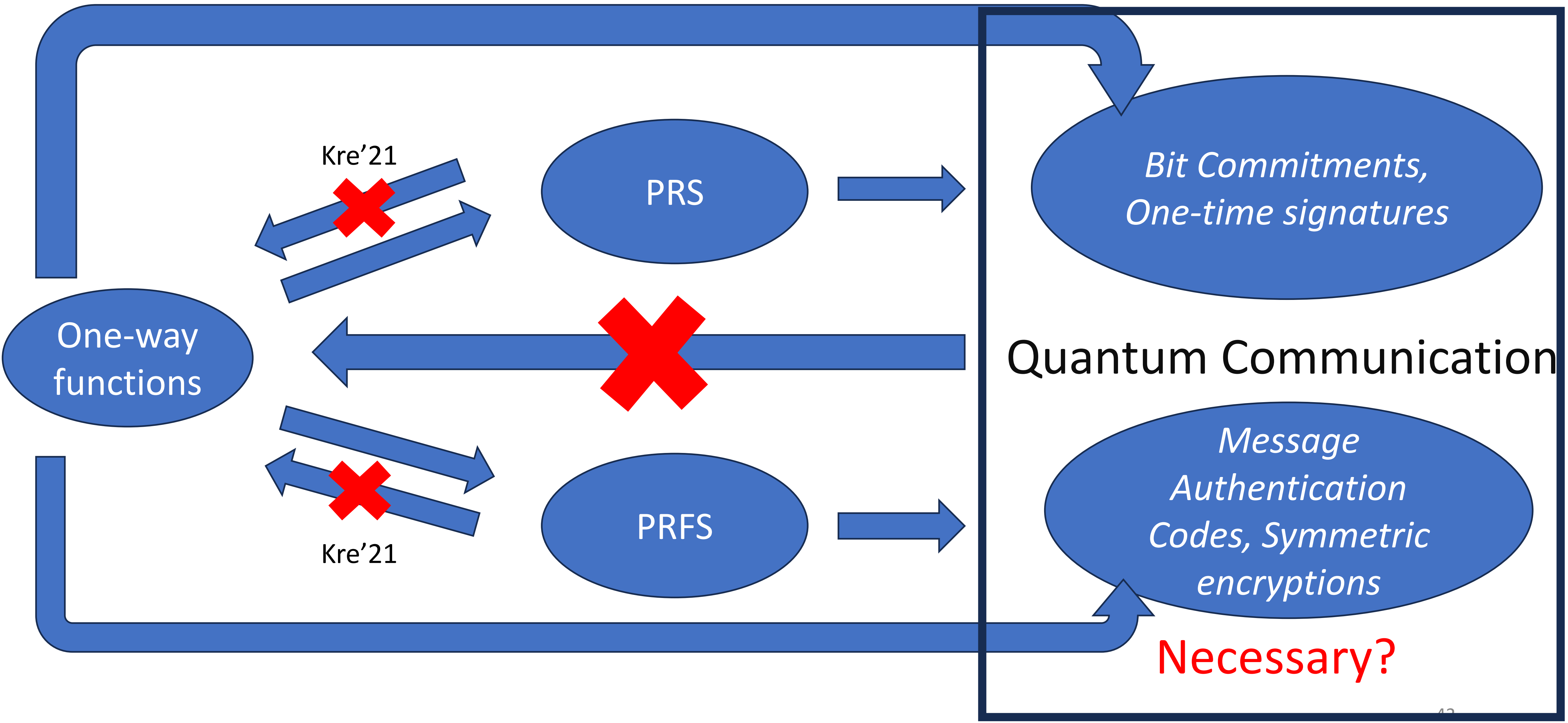


Concurrent works

- Short output PRS. (Ananth-Gulati-Qian-Yuen'22)
- Pseudo-deterministic PRG. (Ananth-Lin-Yuen'23)

Why do we care?



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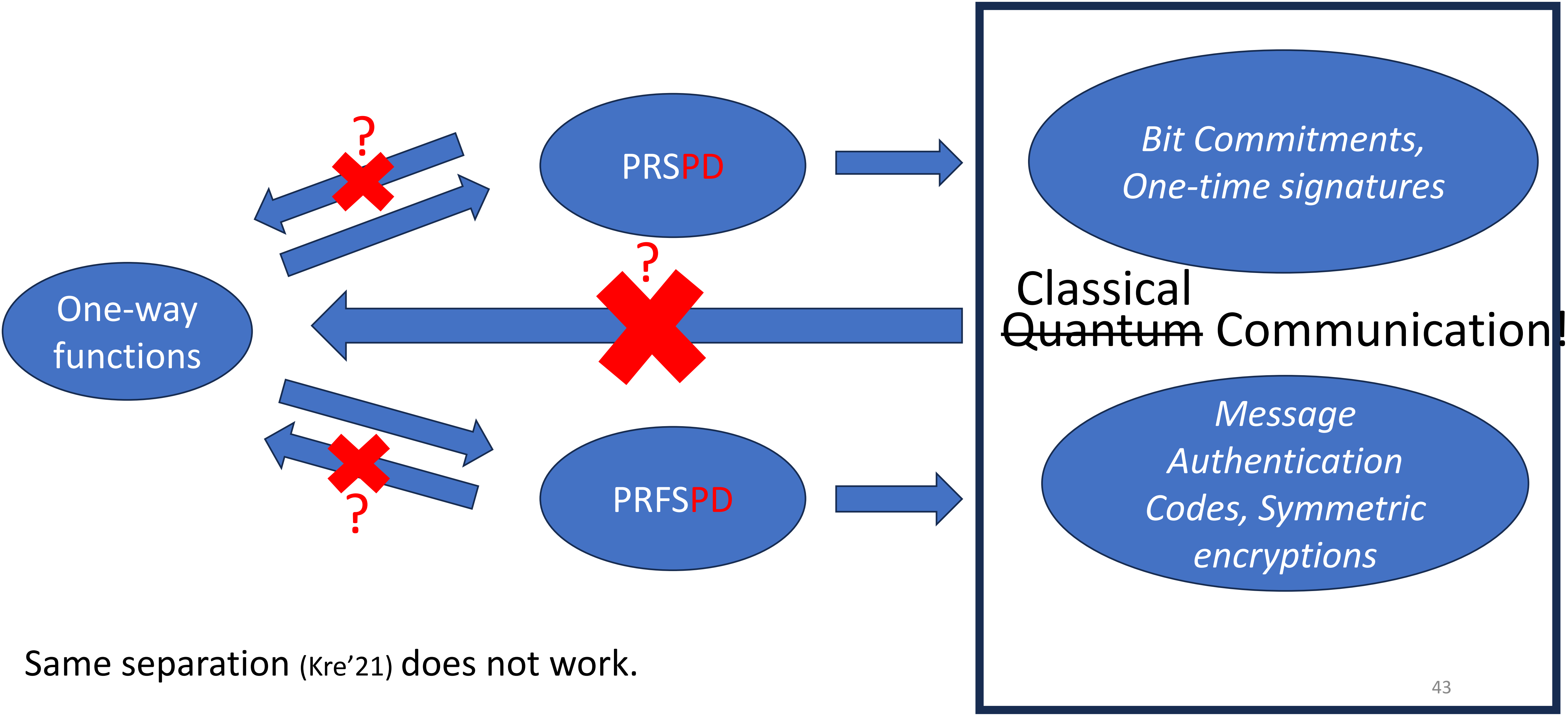
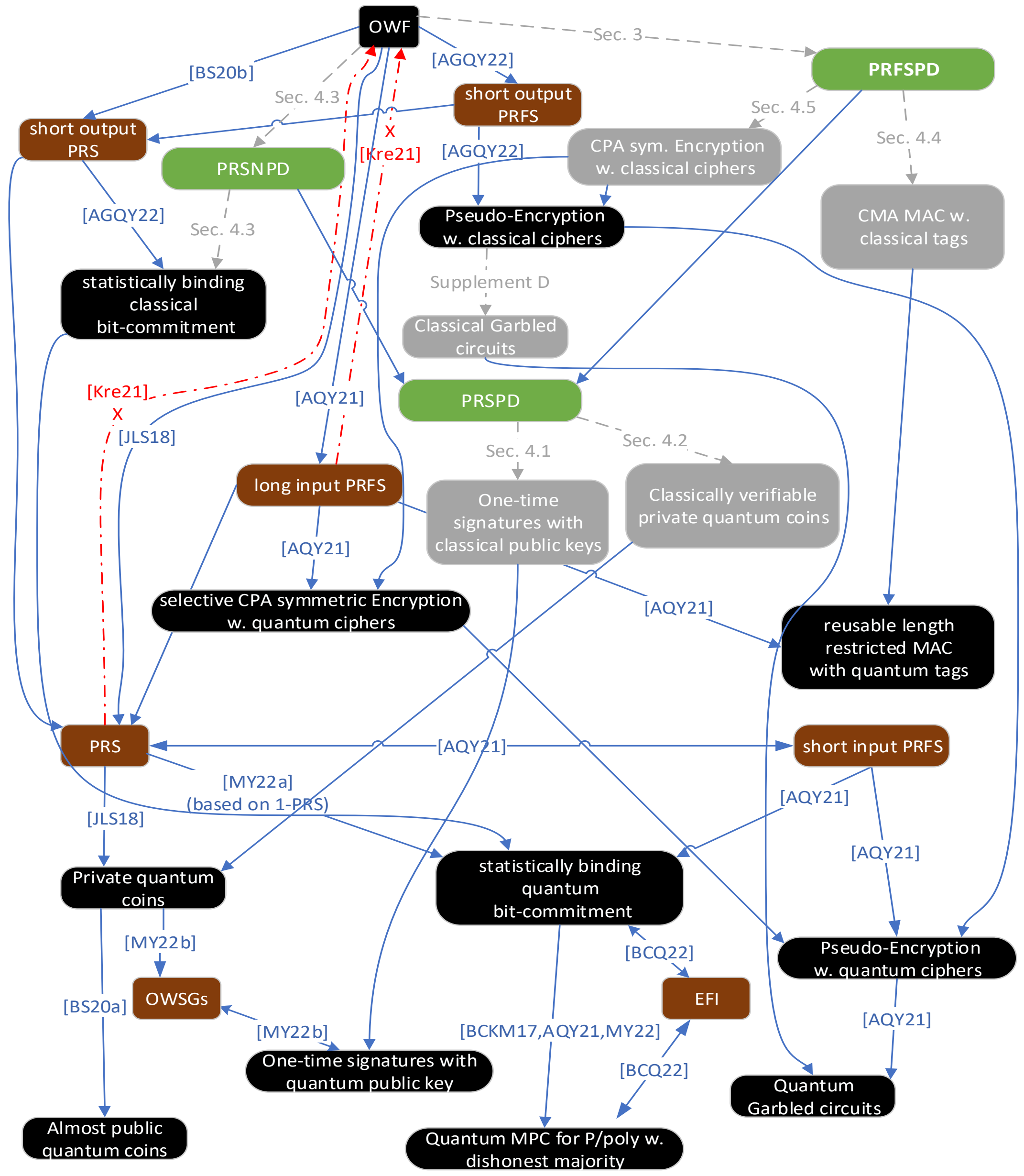


Same separation (Kre'21) does not work.

Full picture currently

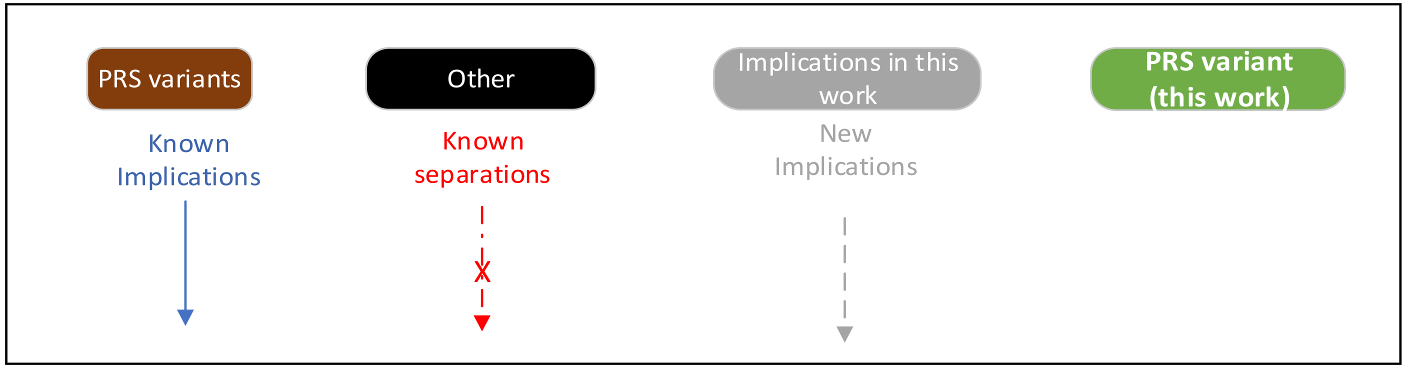


Welcome to the Jungle!

https://sattath.github.io/qcrypto-graph/

Simplify?

Legend



Open-problem

PRS (or PRFS) $\not\Rightarrow$ One-way functions (Kre'21).

PRSPD (or PRFSPD) $\overset{?}{\not\Rightarrow}$ One-way functions.

Separation $\Longrightarrow$ Most classical minicrypt primitives do not need one-way functions or quantum communication.

Related question: Separation of short-output PRS from OWF?

Template for the applications

Template for dequantazing PRS/PRFS applications

Quantum communication

k

PRS/PRFS state

$m, |\psi_{k,m}\rangle$

k

$MAC.Verify_k\,(m)$

$|\psi\rangle \leftarrow PRFS.Gen\,(k, m)$

✔ ❌ $\leftarrow SWAP(|\psi\rangle, |\psi_{k,m}\rangle)$

$MAC.Sign(k, m)$

$|\psi_{k,m}\rangle \leftarrow PRFS.Gen(k, m)$

Template for dequantazing PRS/PRFS applications

Classical

- ~~Quantum~~ communication

k

Proof of destruction

of the PRS/PRFS state

m, p

k

$MAC.Verify_k\,(p, m)$

✔ ✘ $\leftarrow PRFS.Verify_k\,(p, m)$

$MAC.Sign(k, m)$

$|\psi_{k,m}\rangle \leftarrow PRFS.Gen(k, m)$

$p \leftarrow PRFS.Destruct\,(k, m)$

Works in most but not in all cases!

Thank you!

Challenges in this template

One-way functions → One-time signatures (Lam79)

y

k'

$OWF(k') = y?$

$k \leftarrow Uniform$

$y \leftarrow \boxed{\text{OWF}} \leftarrow k$

Public key of the signature are OWF images (like y)

Challenges in this template

PRS ➡ One-time signatures (MY22)

$|\psi_k\rangle$

k'

$Gen(k') \approx |\psi_k\rangle$?

$k \leftarrow Uniform$

$|\psi_k\rangle \leftarrow \boxed{\text{Gen}} \leftarrow k$

Public key of the signature are PRS states (like $|\psi_k\rangle$)

Challenges in this template

PRSPD ⇒ One-time signatures (Morimae-Yamakawa'22)

p_k

k'

$Ver_{k'}(p_k)$=1?

$k \leftarrow Uniform$

$p_k \leftarrow$ Destruct·Gen $\leftarrow k$

- Pseudorandomness/unforgeability is not enough!
- Add a dummy key $\tilde{k}$ that accepts Ver on all proofs.
  - All previous security guarantees hold!
  - Adversary can output $\tilde{k}$ trivially.

Proofs of destruction are not one-way!

Solution: Change the verification algorithm to rule out dummy keys!

Thank you!

Example of an application: MAC construction

Hurdles in finding a separation

Other studied variants of Pseudorandom states

- PRFS (potentially stronger than PRS)
- Short Output PRS (potentially stronger)
- EFI (potentially weaker)
- One-way states (potentially weaker)